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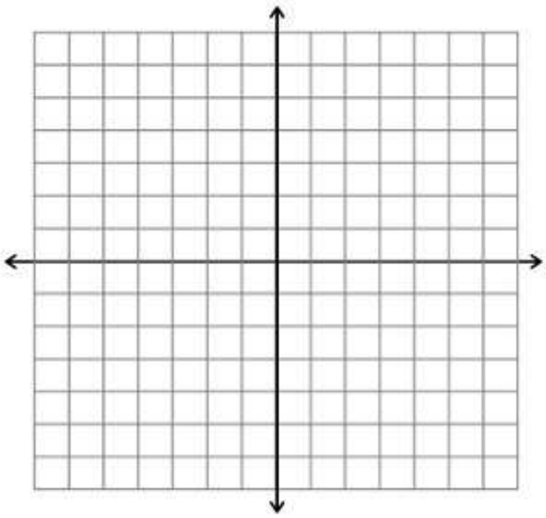
**Math 141 Winter 2015** Name:_____________________

Final

1. Start with graph $f(x) = x^3$. Move this graph 3 units to the right, reflect it over the x-axis, and move it down 2 units. Write down the equation of your new function and sketch.



New $f(x) = -(x+3)^3 - 2$

2. Answer the following questions regarding slope.
    a. The slope between two points is defined as $m = \frac{y_2 - y_1}{x_2 - x_1}$. If you have two points $P1\ (x, f(x))$ and $P2\ \left(x + h, f(x + h)\right)$. Find the slope between them. What is this formula?

    *The slope between these two points is the difference quotient. See any of the quizzes for the exact answer.*

    b. If $f(x) = 3x^2 + 2x - 1$, use this in the formula you found in part a and simplify.

    *6x + 2 +3h*

3. The electric current I, with units in amperes, in a circuit varies directly as the voltage V. When 12 volts are applied, the current is 4 amperes. What is the current when 18 volts are applied?

    *I = kV so k=1/3 and I=6*

4. Consider the following rational function $f(x) = \frac{x^2 - x - 6}{x^3 + 3x^2 - x - 3}$.
    a. State the domain of the function

    $x \neq 1, -1, -3$

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b. What are the x & y intercepts?

*x int- (-2, 0) and (3,0)*

*y- int (0, 2)*

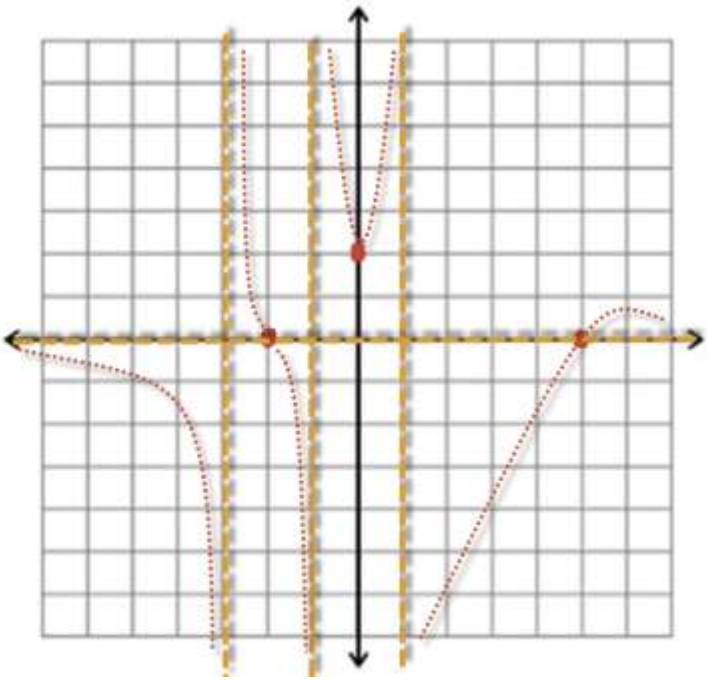
c. What are the *equations* for the horizontal/oblilque and vertical asymptotes?

*VA: x = 1, x=-1, x= -3*

*HA: y = 0 (degree of denominator is larger than degree of numerator)*

d. The horizontal asymptotes come from analyzing the ____END___ behavior of the function. In other, words as $x \rightarrow$ _$\pm\infty$___.

e. Sketch the graph. Be sure to label all intercepts and asymptotes.



5. Let $f(x) = x^3 + x^2 - 7x - 15$. One of the roots of this function is known to be $x = 3$.

a. Use synthetic or long division to find remaining factor. What kind of factor is it? (Hint: it's either a linear factor or irreducible quadratic)

$(x-3)(x^2+4x+5)$, *irreducible quadratic*

b. Use the discriminant to determine what kind of zeros remain.

$b^2 - 4ac = 16 - 20 = -4 < 0$ *therefore there are only imaginary zeros left*

c. What are the remaining zeros? Use the quadratic equation to help you.

$$\frac{-4 \pm \sqrt{-4}}{2} = -2 \pm i$$

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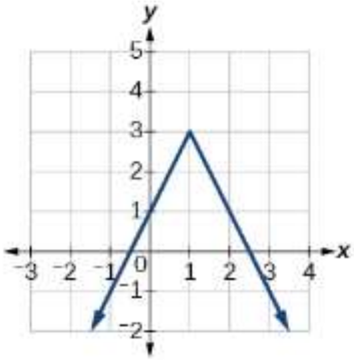
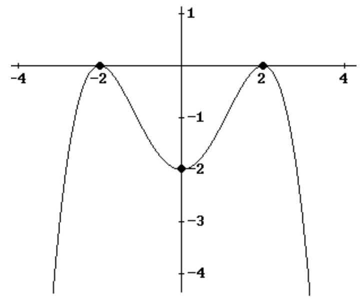
6. Find an equation for each of the following functions below:

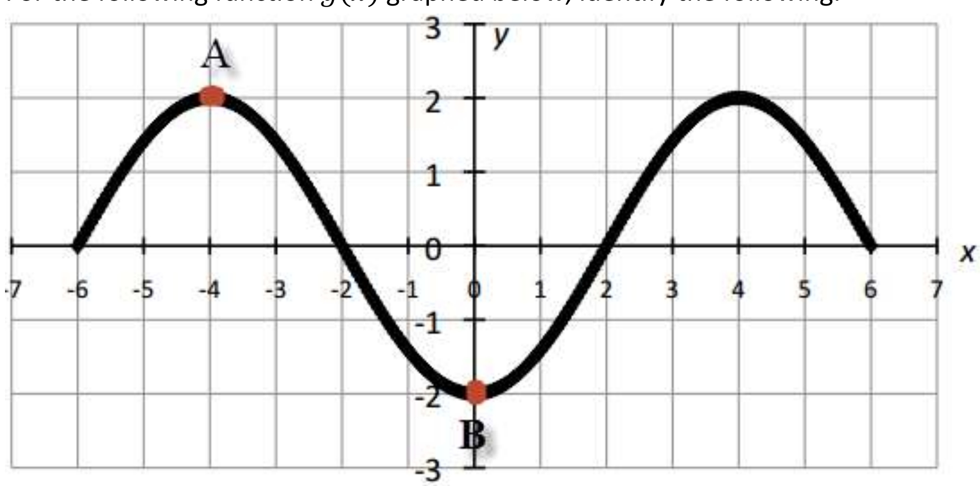



$f(x) = -|2(x-1)| + 3$



$f(x) = -\frac{1}{8}(x+2)^2(x-2)^2$

7. For the following function $g(x)$ graphed below, identify the following:




a. What is the domain of $g(x)$? __-6 <= x + 6________

b. What is the range of $g(x)$? ____-2<= y <= 2____________

c. What are the real zeros of the function? (List as coordinate pairs)
_ (-6,0), (-2,0), (2,0), (6,0) ______________________________

d. What are the local maximums? _ (-4, 2), (4,2) _________

e. What is/are the the local minimum(s)? __ (0,-2) ___________

f. Where is this function decreasing? _on the interval (-4, 0)U(4,6)_______

g. What is/are the absolute maximum(s) ? ____(-4, 2), (4,2) _________

h. Is $g(x)$ odd, even or neither? ____*EVEN*_________

i. Is this function one-to-one?____*NO*_____________

j. If you said no above, how could you limit the domain to make it one-to-one?___*more than one answer is possible 0<= x <= 4*_______________

k. Find the secant line that passes through points A & B. _____*y = -x -2*____________________

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8. Use the intermediate value theorem to prove there is at least one real zero between $t = 1$ and $t = 2$ for the function $h(t) = -t^3 + 2t + 1$

   $h(1) = 2$ *which is* $> 0$ *AND* $h(2) = -3$ *which is* $< 0$ *because there is a sign switch there must be a zero*

9. The probability of a car arriving at Taco Bell $t$ minutes after 11 pm is given by the function $p(t) = 1 - e^{-0.25t}$

   a. Calculate $p(30)$.

   .9995

   b. What value does $p(t)$ approach, and why? Use your exponential rules to help justify your answer.

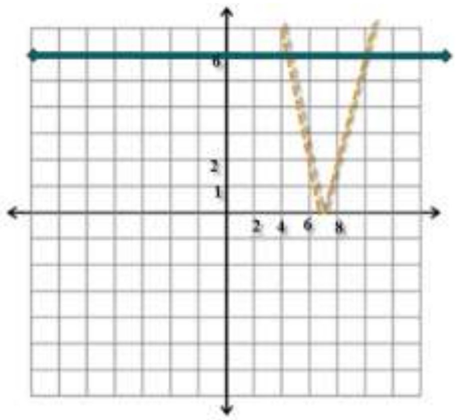
   *It approaches 1 because* $e^{-x} = \frac{1}{e^x}$. *As x gets very big* $\frac{1}{e^x}$ *will become very small, so it will almost be 1 -0*
   *We don't care about negative time given the context of the application (we can only have positive time after a 11pm.*

10. Let $f(x) = 2|x - 7|$ and $g(x) = 6$.

    a. Solve $f(x) > g(x)$

    x>10 or x < 4

    b. Sketch $f(x)$ & $g(x)$ on the same plane. Label three points on $f(x)$.




11. The half-life of 226-Ra (Radon) is 1620 years. Assume an exponential model of $A = A_0e^{kt}$.

    a. In this problem, do you expect $k$ to be positive or negative and why?

    *we expect it to be negative since we have decay over time*

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b. Solve for $k$ and write out the function for A.

*$k$ = -.000427*

c. Solve for the inverse of the equation you found in part b.

$t = \frac{\ln\left(\frac{A}{A_0}\right)}{k}$

d. Using the equation from part c, find how much time it will take for your sample of Radon to reduce to 75%.

$t = \frac{\ln(0.75)}{-.000427}$ = 674 years

12. Let $f(x) = 3x - 1$ and $g(x) = \sqrt{x-2}$

a. Find $g(f(x))$

$g(x) = \sqrt{(3x-1)-2}$ = $\sqrt{3x-3}$

b. What is the domain of $g(f(x))$?

$x \geq 1$

c. What is $(g - f)(11)$?

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d. What is $g^{-1}(x)$?

$g^{-1}(x) = x^2 + 2$ for $x \geq 0$

$NOTE\ THE\ DOMAIN\ AND\ RANGE\ HERE\ FOR\ BOTH\ THE\ FUNCTION\ AND\ THE\ INVERSE$

e. What is $g(g^{-1}(x))$? X

13. Expand the following logarithm as far as possible $\log_2(\frac{x^2\sqrt{y}}{4z})$.

$2\log_2 x + \frac{1}{2}\log_2 y - 2 - \log_2 z$

14. An object is launched at 19.6 m/s from a 58.8m tall building. The equation for the objects height $s$ at time $t$, is given by the formula $s(t) = -4.9t^2 + 19.6t + 58.8$.

a. What is the height of the object after $t = 2$?

78.4

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b. When does the object strike the ground? (Hint: you should wind up with two solutions, but one of them will be extraneous in regards to the context of the problem)
-4.9(t-6)(t+2) so my two solutions are t = -2 (this is extraneous because it deals with negative time) and t=6. So the final answer is t=6

c. At what time does the object reach its maximum height?
t = 2

d. What is the maximum height that it reaches?
78.4

e. What is the axis of symmetry for the function above?
t = 2

15. Consider a right circular cone with diameter of 12 cm for the base and altitude of 16 cm. Suppose the cone is held vertex down. The volume of a right circular cone is given as $V = \frac{1}{3}\pi r^2 h$.

a. What is the volume of the entire cone?
V = 192 $\pi$

b. Using your knowledge of similar triangles, write an expression for the radius *r,* as a function of height *h.*

$$\frac{6}{r} = \frac{16}{h}, so\ r = \frac{6h}{16} = \frac{3h}{8}$$

c. If the cone is held with its vertex down and water placed into it until the cone was half way full, what would be the depth of the water? (i.e. what would the height be?) You will need both of the equations above to help you answer this question.

$$96\pi = \frac{1}{3}\pi\left(\frac{3h}{8}\right)^2 h \text{ so } h = 12.7$$