9.1 Exercises

In Exercises 1-10, complete each of the following tasks.

i. Set up a coordinate system on a sheet of graph paper. Label and scale each axis.

ii. Complete the table of points for the given function. Plot each of the points on your coordinate system, then use them to help draw the graph of the given function.

iii. Use different colored pencils to project all points onto the $x$- and $y$-axes to determine the domain and range. Use interval notation to describe the domain of the given function.

1. $f(x) = -\sqrt{x}$

2. $f(x) = \sqrt{-x}$

3. $f(x) = \sqrt{x + 2}$

4. $f(x) = \sqrt{5 - x}$

5. $f(x) = \sqrt{x} + 2$

6. $f(x) = \sqrt{x} - 1$

7. $f(x) = \sqrt{x + 3} + 2$

8. $f(x) = \sqrt{x - 1} + 3$

9. $f(x) = \sqrt{3 - x}$

10. $f(x) = -\sqrt{x + 3}$

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Version: Fall 2007
In Exercises 11-20, perform each of the following tasks.

i. Set up a coordinate system on a sheet of graph paper. Label and scale each axis. Remember to draw all lines with a ruler.

ii. Use geometric transformations to draw the graph of the given function on your coordinate system without the use of a graphing calculator. Note: You may check your solution with your calculator, but you should be able to produce the graph without the use of your calculator.

iii. Use different colored pencils to project the points on the graph of the function onto the $x$- and $y$-axes. Use interval notation to describe the domain and range of the function.

11. $f(x) = \sqrt{x} + 3$

12. $f(x) = \sqrt{x} + 3$

13. $f(x) = \sqrt{x} - 2$

14. $f(x) = \sqrt{x} - 2$

15. $f(x) = \sqrt{x} + 3 + 1$

16. $f(x) = \sqrt{x} - 2 - 1$

17. $f(x) = -\sqrt{x} + 4$

18. $f(x) = -\sqrt{x} + 4$

19. $f(x) = -\sqrt{x} + 3$

20. $f(x) = -\sqrt{x} + 3$

21. To draw the graph of the function $f(x) = \sqrt{3 - x}$, perform each of the following steps in sequence without the aid of a calculator.

i. Set up a coordinate system and sketch the graph of $y = \sqrt{x}$. Label the graph with its equation.

ii. Set up a second coordinate system and sketch the graph of $y = \sqrt{-x}$. Label the graph with its equation.

iii. Set up a third coordinate system and sketch the graph of $y = \sqrt{(x - 3)}$. Label the graph with its equation. This is the graph of $f(x) = \sqrt{3 - x}$. Use interval notation to state the domain and range of this function.

22. To draw the graph of the function $f(x) = \sqrt{-x} - 3$, perform each of the following steps in sequence.

i. Set up a coordinate system and sketch the graph of $y = \sqrt{x}$. Label the graph with its equation.

ii. Set up a second coordinate system and sketch the graph of $y = \sqrt{-x}$. Label the graph with its equation.

iii. Set up a third coordinate system and sketch the graph of $y = \sqrt{(x + 3)}$. Label the graph with its equation. This is the graph of $f(x) = \sqrt{-x} - 3$. Use interval notation to state the domain and range of this function.

23. To draw the graph of the function $f(x) = \sqrt{-x} - 1$, perform each of the following steps in sequence without the aid of a calculator.

i. Set up a coordinate system and sketch the graph of $y = \sqrt{x}$. Label the graph with its equation.

ii. Set up a second coordinate system and sketch the graph of $y = \sqrt{-x}$. Label the graph with its equation.

iii. Set up a third coordinate system and sketch the graph of $y = \sqrt{(x + 1)}$. Label the graph with its equation. This is the graph of $f(x) = \sqrt{-x} - 1$. Use interval notation to state the domain and range of this function.
24. To draw the graph of the function \( f(x) = \sqrt{1-x} \), perform each of the following steps in sequence.

i. Set up a coordinate system and sketch the graph of \( y = \sqrt{x} \). Label the graph with its equation.

ii. Set up a second coordinate system and sketch the graph of \( y = \sqrt{-x} \). Label the graph with its equation.

iii. Set up a third coordinate system and sketch the graph of \( y = \sqrt{-(x-1)} \). Label the graph with its equation. This is the graph of \( f(x) = \sqrt{1-x} \). Use interval notation to state the domain and range of this function.

In Exercises 29-40, find the domain of the given function algebraically.

29. \( f(x) = \sqrt{2x+9} \)

30. \( f(x) = \sqrt{-3x+3} \)

31. \( f(x) = \sqrt{-8x-3} \)

32. \( f(x) = \sqrt{-3x+6} \)

33. \( f(x) = \sqrt{-6x-8} \)

34. \( f(x) = \sqrt{8x-6} \)

35. \( f(x) = \sqrt{-7x+2} \)

36. \( f(x) = \sqrt{8x-3} \)

37. \( f(x) = \sqrt{6x+3} \)

38. \( f(x) = \sqrt{x-5} \)

39. \( f(x) = \sqrt{-7x-8} \)

40. \( f(x) = \sqrt{7x+8} \)

In Exercises 25-28, perform each of the following tasks.

i. Draw the graph of the given function with your graphing calculator. Copy the image in your viewing window onto your homework paper. Label and scale each axis with xmin, xmax, ymin, and ymax. Label your graph with its equation. Use the graph to determine the domain of the function and describe the domain with interval notation.

ii. Use a purely algebraic approach to determine the domain of the given function. Use interval notation to describe your result. Does it agree with the graphical result from part (i)?

25. \( f(x) = \sqrt{2x+7} \)

26. \( f(x) = \sqrt{7-2x} \)

27. \( f(x) = \sqrt{12-4x} \)

28. \( f(x) = \sqrt{12+2x} \)
9.1 Answers

1. Domain = $[0, \infty)$, Range = $(-\infty, 0]$.

\[ f(x) = \sqrt{-x} \]

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>4</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>0</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
</tr>
</tbody>
</table>

3. Domain = $[-2, \infty)$, Range = $[0, \infty)$.

\[ f(x) = \sqrt{x+2} \]

<table>
<thead>
<tr>
<th>$x$</th>
<th>-2</th>
<th>-1</th>
<th>2</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

5. Domain = $[0, \infty)$, Range = $[2, \infty)$.

\[ f(x) = \sqrt{x+2} \]

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>4</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

7. Domain = $[-3, \infty)$, Range = $[2, \infty)$.

\[ f(x) = \sqrt{x+3}+2 \]

<table>
<thead>
<tr>
<th>$x$</th>
<th>-3</th>
<th>-2</th>
<th>1</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>
9. Domain = \((-\infty, 3]\), Range = \([0, \infty)\).

\[
\begin{array}{c|cccc}
  x & -6 & -1 & 2 & 3 \\
  f(x) & 3 & 2 & 1 & 0 \\
\end{array}
\]

11. Domain = \([0, \infty)\), Range = \([3, \infty)\).

13. Domain = \([2, \infty)\), Range = \([0, \infty)\).

15. Domain = \([-5, \infty)\), Range = \([1, \infty)\).

17. Domain = \([-4, \infty)\), Range = \((-\infty, 0]\).

19. Domain = \([0, \infty)\), Range = \((-\infty, 3]\).
21. Domain = \((-\infty, 3]\), Range = \([0, \infty)\).

\[ f(x) = \sqrt{3-x} \]

23. Domain = \((-\infty, -1]\), Range = \([0, \infty)\).

\[ f(x) = \sqrt{-x-1} \]

25. Domain = \([-7/2, \infty)\)

\[ f(x) = \sqrt{2x+7} \]

27. Domain = \((-\infty, 3]\)

\[ f(x) = \sqrt{12-4x} \]

29. \((-9/7, \infty)\)

31. \((-\infty, -3/8]\)

33. \((-\infty, -4/3]\)

35. \((-\infty, 2/7]\)

37. \([-1/2, \infty)\)

39. \((-\infty, -8/7]\)
9.2 Exercises

1. Use a calculator to first approximate $\sqrt{5} \sqrt{2}$. On the same screen, approximate $\sqrt{10}$. Report the results on your homework paper.

2. Use a calculator to first approximate $\sqrt{7} \sqrt{10}$. On the same screen, approximate $\sqrt{70}$. Report the results on your homework paper.

3. Use a calculator to first approximate $\sqrt{3} \sqrt{11}$. On the same screen, approximate $\sqrt{33}$. Report the results on your homework paper.

4. Use a calculator to first approximate $\sqrt{5} \sqrt{3}$. On the same screen, approximate $\sqrt{65}$. Report the results on your homework paper.

In Exercises 5-20, place each of the radical expressions in simple radical form. As in Example 3 in the narrative, check your result with your calculator.

5. $\sqrt{18}$
6. $\sqrt{80}$
7. $\sqrt{112}$
8. $\sqrt{72}$
9. $\sqrt{108}$
10. $\sqrt{54}$
11. $\sqrt{50}$
12. $\sqrt{48}$
13. $\sqrt{245}$

14. $\sqrt{150}$
15. $\sqrt{98}$
16. $\sqrt{252}$
17. $\sqrt{45}$
18. $\sqrt{294}$
19. $\sqrt{24}$
20. $\sqrt{32}$

In Exercises 21-26, use prime factorization (as in Examples 10 and 11 in the narrative) to assist you in placing the given radical expression in simple radical form. Check your result with your calculator.

21. $\sqrt{2016}$
22. $\sqrt{2700}$
23. $\sqrt{14175}$
24. $\sqrt{44000}$
25. $\sqrt{20250}$
26. $\sqrt{3564}$

In Exercises 27-46, place each of the given radical expressions in simple radical form. Make no assumptions about the sign of the variables. Variables can either represent positive or negative numbers.

27. $\sqrt{(6x - 11)^4}$

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28. \( \sqrt{16h^8} \)

29. \( \sqrt{25f^2} \)

30. \( \sqrt{25j^8} \)

31. \( \sqrt{16m^2} \)

32. \( \sqrt{25a^2} \)

33. \( \sqrt{(7x + 5)^{12}} \)

34. \( \sqrt{9w^{10}} \)

35. \( \sqrt{25x^2 - 50x + 25} \)

36. \( \sqrt{49x^2 - 42x + 9} \)

37. \( \sqrt{25x^2 + 90x + 81} \)

38. \( \sqrt{25g^{14}} \)

39. \( \sqrt{(3x + 6)^{12}} \)

40. \( \sqrt{(9x - 8)^{12}} \)

41. \( \sqrt{36x^2 + 36x + 9} \)

42. \( \sqrt{4e^2} \)

43. \( \sqrt{4p^{10}} \)

44. \( \sqrt{25x^{12}} \)

45. \( \sqrt{25q^6} \)

46. \( \sqrt{16h^{12}} \)

47. Given that \( x < 0 \), place the radical expression \( \sqrt{32x^6} \) in simple radical form. Check your solution on your calculator for \( x = -2 \).

48. Given that \( x < 0 \), place the radical expression \( \sqrt{54x^5} \) in simple radical form. Check your solution on your calculator for \( x = -2 \).

49. Given that \( x < 0 \), place the radical expression \( \sqrt{27x^{12}} \) in simple radical form. Check your solution on your calculator for \( x = -2 \).

50. Given that \( x < 0 \), place the radical expression \( \sqrt{44x^{10}} \) in simple radical form. Check your solution on your calculator for \( x = -2 \).

In Exercises 51-54, follow the lead of Example 17 in the narrative to simplify the given radical expression and check your result with your graphing calculator.

51. Given that \( x < 4 \), place the radical expression \( \sqrt{x^2 - 8x + 16} \) in simple radical form. Use a graphing calculator to show that the graphs of the original expression and your simple radical form agree for all values of \( x \) such that \( x < 4 \).

52. Given that \( x \geq -2 \), place the radical expression \( \sqrt{x^2 + 4x + 4} \) in simple radical form. Use a graphing calculator to show that the graphs of the original expression and your simple radical form agree for all values of \( x \) such that \( x \geq -2 \).

53. Given that \( x \geq 5 \), place the radical expression \( \sqrt{x^2 - 10x + 25} \) in simple radical form. Use a graphing calculator to show that the graphs of the original expression and your simple radical form agree for all values of \( x \) such that \( x \geq 5 \).

54. Given that \( x < -1 \), place the radical expression \( \sqrt{x^2 + 2x + 1} \) in simple radical form. Use a graphing calculator to show that the graphs of the original expression and your simple radical form agree for all values of \( x \) such that \( x < -1 \).
In Exercises 55-72, place each radical expression in simple radical form. Assume that all variables represent positive numbers.

55. $\sqrt{9d^{13}}$
56. $\sqrt{4k^2}$
57. $\sqrt{25x^2 + 40x + 16}$
58. $\sqrt{9x^2 - 30x + 25}$
59. $\sqrt{4j^{11}}$
60. $\sqrt{16j^6}$
61. $\sqrt{25m^2}$
62. $\sqrt{9e^9}$
63. $\sqrt{4e^5}$
64. $\sqrt{25z^2}$
65. $\sqrt{25h^{10}}$
66. $\sqrt{25b^2}$
67. $\sqrt{9s^7}$
68. $\sqrt{9e^7}$
69. $\sqrt{4f^8}$
70. $\sqrt{9d^{15}}$
71. $\sqrt{9q^{10}}$
72. $\sqrt{4w^7}$

In Exercises 73-80, place each given radical expression in simple radical form. Assume that all variables represent positive numbers.

73. $\sqrt{2f^5} \cdot \sqrt{8f^3}$
74. $\sqrt{3s^3} \cdot \sqrt{243s^3}$
75. $\sqrt{2k^7} \cdot \sqrt{32k^3}$
76. $\sqrt{2n^9} \cdot \sqrt{8n^3}$
77. $\sqrt{2e^9} \cdot \sqrt{8e^3}$
78. $\sqrt{5n^9} \cdot \sqrt{125n^3}$
79. $\sqrt{3z^5} \cdot \sqrt{27z^3}$
80. $\sqrt{3t^7} \cdot \sqrt{27t^3}$
9.2 Answers

1. \( \sqrt{10} \approx 3.16227766 \)

3. \( \sqrt{33} \approx 5.744562647 \)

5. \( 3\sqrt{2} \)

7. \( 4\sqrt{7} \)

9. \( 6\sqrt{3} \)

11. \( 5\sqrt{2} \)

13. \( 7\sqrt{5} \)

15. \( 7\sqrt{2} \)

17. \( 3\sqrt{5} \)

19. \( 2\sqrt{6} \)

21. \( 12\sqrt{14} \)

23. \( 45\sqrt{7} \)

25. \( 45\sqrt{10} \)

27. \( (6x - 11)^2 \)

29. \( 5|f| \)

31. \( 4|m| \)

33. \( (7x + 5)^6 \)

35. \( |5x - 5| \)

37. \( |5x + 9| \)

39. \( (3x + 6)^6 \)

41. \( |6x + 3| \)

43. \( 2p^4|p| \)

45. \( 5q^2|q| \)

47. \( -4x^3\sqrt{2} \)

49. \( 3x^6\sqrt{3} \)
51. \(-x + 4\). The graphs of \(y = -x + 4\) and \(y = \sqrt{x^2 - 8x + 16}\) follow. Note that they agree for \(x < 4\).

53. \(x - 5\). The graphs of \(y = x - 5\) and \(y = \sqrt{x^2 - 10x + 25}\) follow. Note that they agree for \(x \geq 5\).

55. \(3d^6\sqrt{d}\)

57. \(5x + 4\)

59. \(2j^5\sqrt{j}\)

61. \(5m\)

63. \(2e^2\sqrt{e}\)

65. \(5h^5\)

67. \(3s^3\sqrt{s}\)
9.3 Exercises

1. Use a calculator to first approximate $\sqrt{5}/\sqrt{2}$. On the same screen, approximate $\sqrt{5}/2$. Report the results on your homework paper.

2. Use a calculator to first approximate $\sqrt{7}/\sqrt{5}$. On the same screen, approximate $\sqrt{7}/5$. Report the results on your homework paper.

3. Use a calculator to first approximate $\sqrt{12}/\sqrt{2}$. On the same screen, approximate $\sqrt{6}$. Report the results on your homework paper.

4. Use a calculator to first approximate $\sqrt{15}/\sqrt{5}$. On the same screen, approximate $\sqrt{3}$. Report the results on your homework paper.

5. In Exercises 5-16, place each radical expression in simple radical form. As in Example 2 in the narrative, check your result with your calculator.

   5. $\sqrt{\frac{3}{8}}$
   6. $\sqrt{\frac{5}{12}}$
   7. $\sqrt{\frac{11}{20}}$
   8. $\sqrt{\frac{3}{2}}$
   9. $\sqrt{\frac{11}{18}}$
   10. $\sqrt{\frac{7}{5}}$
   11. $\sqrt{\frac{4}{3}}$
   12. $\sqrt{\frac{16}{5}}$
   13. $\sqrt{\frac{49}{12}}$
   14. $\sqrt{\frac{81}{20}}$
   15. $\sqrt{\frac{100}{7}}$
   16. $\sqrt{\frac{36}{5}}$

In Exercises 17-28, place each radical expression in simple radical form. As in Example 4 in the narrative, check your result with your calculator.

   17. $\frac{1}{\sqrt{12}}$
   18. $\frac{1}{\sqrt{8}}$
   19. $\frac{1}{\sqrt{20}}$
   20. $\frac{1}{\sqrt{27}}$
   21. $\frac{6}{\sqrt{8}}$
   22. $\frac{4}{\sqrt{12}}$

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23. \( \frac{5}{\sqrt{20}} \)
24. \( \frac{9}{\sqrt{27}} \)
25. \( \frac{6}{2\sqrt{3}} \)
26. \( \frac{10}{3\sqrt{5}} \)
27. \( \frac{15}{2\sqrt{20}} \)
28. \( \frac{3}{2\sqrt{18}} \)

In Exercises 29-36, place the given radical expression in simple form. Use prime factorization as in Example 8 in the narrative to help you with the calculations. As in Example 6, check your result with your calculator.

29. \( \frac{1}{\sqrt{96}} \)
30. \( \frac{1}{\sqrt{432}} \)
31. \( \frac{1}{\sqrt{250}} \)
32. \( \frac{1}{\sqrt{108}} \)
33. \( \sqrt{\frac{5}{96}} \)
34. \( \sqrt{\frac{2}{135}} \)
35. \( \sqrt{\frac{2}{1485}} \)
36. \( \sqrt{\frac{3}{280}} \)

In Exercises 37-44, place each of the given radical expressions in simple radical form. Make no assumptions about the sign of any variable. Variables can represent either positive or negative numbers.

37. \( \sqrt{\frac{8}{x^4}} \)
38. \( \sqrt{\frac{12}{x^6}} \)
39. \( \sqrt{\frac{20}{x^2}} \)
40. \( \sqrt{\frac{32}{x^{14}}} \)
41. \( \frac{2}{\sqrt{8x^8}} \)
42. \( \frac{3}{\sqrt{12x^6}} \)
43. \( \frac{10}{\sqrt{20x^{10}}} \)
44. \( \frac{12}{\sqrt{6x^4}} \)

In Exercises 45-48, follow the lead of Example 8 in the narrative to craft a solution.

45. Given that \( x < 0 \), place the radical expression \( \frac{6}{\sqrt{2x^5}} \) in simple radical form. Check your solution on your calculator for \( x = -1 \).

46. Given that \( x > 0 \), place the radical expression \( \frac{4}{\sqrt{12x^3}} \) in simple radical form. Check your solution on your calculator for \( x = 1 \).
47. Given that $x > 0$, place the radical expression $8/\sqrt{8x^5}$ in simple radical form. Check your solution on your calculator for $x = 1$.

48. Given that $x < 0$, place the radical expression $15/\sqrt{20x^6}$ in simple radical form. Check your solution on your calculator for $x = -1$.

In Exercises 49-56, place each of the radical expressions in simple form. Assume that all variables represent positive numbers.

49. $\sqrt{\frac{12}{x}}$
50. $\sqrt{\frac{18}{x}}$
51. $\sqrt{\frac{50}{x^3}}$
52. $\sqrt{\frac{72}{x^5}}$
53. $\frac{1}{\sqrt{50x}}$
54. $\frac{2}{\sqrt{18x}}$
55. $\frac{3}{\sqrt{27x^3}}$
56. $\frac{5}{\sqrt{10x^5}}$
9.3 Answers

1. $\sqrt{5}/\sqrt{2}$

3. $\sqrt{12}/\sqrt{2}$

5. $\sqrt{6}/4$

7. $\sqrt{55}/10$

9. $\sqrt{22}/6$

11. $2\sqrt{3}/3$

13. $7\sqrt{3}/6$

15. $10\sqrt{7}/7$

17. $\sqrt{3}/6$

19. $\sqrt{5}/10$

21. $3\sqrt{2}/2$

23. $\sqrt{5}/2$

25. $\sqrt{3}$

27. $3\sqrt{5}/4$

29. $\sqrt{6}/24$

31. $\sqrt{10}/50$

33. $\sqrt{30}/24$

35. $\sqrt{330}/495$

37. $2\sqrt{2}/x^2$

39. $2\sqrt{5}/|x|$

41. $\sqrt{2}/(2x^4)$

43. $\sqrt{5}/(x^4|x|)$

45. $-3\sqrt{2}/x^3$

47. $2\sqrt{2x}/x^3$

49. $2\sqrt{3x}/x$

51. $5\sqrt{2x}/x^2$

53. $\sqrt{2x}/(10x)$

55. $\sqrt{3x}/(3x^2)$
9.4 Exercises

In Exercises 1-14, place each of the radical expressions in simple radical form. Check your answer with your calculator.

1. \(2(5\sqrt{7})\)
2. \(-3(2\sqrt{3})\)
3. \(-\sqrt{3}(2\sqrt{5})\)
4. \(\sqrt{2}(3\sqrt{7})\)
5. \(\sqrt{3}(5\sqrt{6})\)
6. \(\sqrt{2}(-3\sqrt{10})\)
7. \((2\sqrt{5})(-3\sqrt{3})\)
8. \((-5\sqrt{2})(-2\sqrt{7})\)
9. \((-4\sqrt{3})(2\sqrt{6})\)
10. \((2\sqrt{5})(-3\sqrt{10})\)
11. \((2\sqrt{3})^2\)
12. \((-3\sqrt{5})^2\)
13. \((-5\sqrt{2})^2\)
14. \((7\sqrt{11})^2\)

In Exercises 15-22, use the distributive property to multiply. Place your final answer in simple radical form. Check your result with your calculator.

15. \(2(3 + \sqrt{5})\)
16. \(-3(4 - \sqrt{7})\)
17. \(2(-5 + 4\sqrt{2})\)
18. \(-3(4 - 3\sqrt{2})\)
19. \(\sqrt{2}(2 + \sqrt{2})\)
20. \(\sqrt{3}(4 - \sqrt{6})\)
21. \(\sqrt{2}(\sqrt{10} + \sqrt{14})\)
22. \(\sqrt{3}(\sqrt{15} - \sqrt{33})\)

In Exercises 23-30, combine like terms. Place your final answer in simple radical form. Check your solution with your calculator.

23. \(-5\sqrt{2} + 7\sqrt{2}\)
24. \(2\sqrt{3} + 3\sqrt{3}\)
25. \(2\sqrt{6} - 8\sqrt{6}\)
26. \(\sqrt{7} - 3\sqrt{7}\)
27. \(2\sqrt{3} - 4\sqrt{2} + 3\sqrt{3}\)
28. \(7\sqrt{5} + 2\sqrt{7} - 3\sqrt{5}\)
29. \(2\sqrt{3} + 5\sqrt{2} - 7\sqrt{3} + 2\sqrt{2}\)
30. \(3\sqrt{11} - 2\sqrt{7} - 2\sqrt{11} + 4\sqrt{7}\)

In Exercises 31-40, combine like terms where possible. Place your final answer in simple radical form. Use your calculator to check your result.

31. \(\sqrt{45} + \sqrt{20}\)
32. \(-4\sqrt{45} - 4\sqrt{20}\)
33. \(2\sqrt{18} - \sqrt{8}\)

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34. \(-\sqrt{20} + 4\sqrt{45}\)
35. \(-5\sqrt{27} + 5\sqrt{12}\)
36. \(3\sqrt{12} - 2\sqrt{27}\)
37. \(4\sqrt{20} + 4\sqrt{45}\)
38. \(-2\sqrt{18} - 5\sqrt{8}\)
39. \(2\sqrt{15} + 5\sqrt{20}\)
40. \(3\sqrt{27} - 4\sqrt{12}\)

In Exercises 41-48, simplify each of the given rational expressions. Place your final answer in simple radical form. Check your result with your calculator.

41. \(\sqrt{2} - \frac{1}{\sqrt{2}}\)
42. \(3\sqrt{3} - \frac{3}{\sqrt{3}}\)
43. \(2\sqrt{2} - \frac{2}{\sqrt{2}}\)
44. \(4\sqrt{5} - \frac{5}{\sqrt{5}}\)
45. \(5\sqrt{2} + \frac{3}{\sqrt{2}}\)
46. \(6\sqrt{3} + \frac{2}{\sqrt{3}}\)
47. \(\sqrt{8} - \frac{12}{\sqrt{2}} - 3\sqrt{2}\)
48. \(\sqrt{27} - \frac{6}{\sqrt{3}} - 5\sqrt{3}\)

In Exercises 49-60, multiply to expand each of the given radical expressions. Place your final answer in simple radical form. Use your calculator to check your result.

49. \((2 + \sqrt{3})(3 - \sqrt{3})\)
50. \((5 + \sqrt{2})(2 - \sqrt{2})\)
51. \((4 + 3\sqrt{2})(2 - 5\sqrt{2})\)
52. \((3 + 5\sqrt{3})(1 - 2\sqrt{3})\)
53. \((2 + 3\sqrt{2})(2 - 3\sqrt{2})\)
54. \((3 + 2\sqrt{5})(3 - 2\sqrt{5})\)
55. \((2\sqrt{3} + 3\sqrt{2})(2\sqrt{3} - 3\sqrt{2})\)
56. \((8\sqrt{2} + \sqrt{5})(8\sqrt{2} - \sqrt{5})\)
57. \((2 + \sqrt{5})^2\)
58. \((3 - \sqrt{2})^2\)
59. \((\sqrt{3} - 2\sqrt{5})^2\)
60. \((2\sqrt{3} + 3\sqrt{2})^2\)

In Exercises 61-68, place each of the given rational expressions in simple radical form by “rationalizing the denominator.” Check your result with your calculator.

61. \(\frac{1}{\sqrt{5} + \sqrt{3}}\)
62. \(\frac{1}{2\sqrt{3} - \sqrt{2}}\)
63. \(\frac{6}{2\sqrt{5} - \sqrt{2}}\)
64. \(\frac{9}{3\sqrt{3} - \sqrt{6}}\)
In Exercises 69–76, use the quadratic formula to find the solutions of the given equation. Place your solutions in simple radical form and reduce your solutions to lowest terms.

69. \(3x^2 - 8x = 5\)
70. \(5x^2 - 2x = 1\)
71. \(5x^2 = 2x + 1\)
72. \(3x^2 - 2x = 11\)
73. \(7x^2 = 6x + 2\)
74. \(11x^2 + 6x = 4\)
75. \(x^2 = 2x + 19\)
76. \(100x^2 = 40x - 1\)

In Exercises 77–80, we will suspend the usual rule that you should rationalize the denominator. Instead, just this one time, rationalize the numerator of the resulting expression.

77. Given \(f(x) = \sqrt{x}\), evaluate the expression
\[
\frac{f(x) - f(2)}{x - 2},
\]
and then “rationalize the numerator.”

78. Given \(f(x) = \sqrt{x + 2}\), evaluate the expression
\[
\frac{f(x) - f(3)}{x - 3},
\]
and then “rationalize the numerator.”

79. Given \(f(x) = \sqrt{x}\), evaluate the expression
\[
\frac{f(x + h) - f(x)}{h},
\]
and then “rationalize the numerator.”

80. Given \(f(x) = \sqrt{x - 3}\), evaluate the expression
\[
\frac{f(x + h) - f(x)}{h},
\]
and then “rationalize the numerator.”
9.4 Answers

1. \(10\sqrt{7}\)

3. \(-2\sqrt{15}\)

5. \(15\sqrt{2}\)

7. \(-6\sqrt{15}\)

9. \(-24\sqrt{2}\)

11. 12

13. 50

15. \(6 + 2\sqrt{5}\)

17. \(-10 + 8\sqrt{2}\)

19. \(2\sqrt{2} + 2\)

21. \(2\sqrt{5} + 2\sqrt{7}\)

23. \(2\sqrt{2}\)

25. \(-6\sqrt{6}\)

27. \(5\sqrt{3} - 4\sqrt{2}\)

29. \(7\sqrt{2} - 5\sqrt{3}\)

31. \(5\sqrt{5}\)

33. \(4\sqrt{2}\)

35. \(-5\sqrt{3}\)

37. \(20\sqrt{5}\)

39. \(16\sqrt{5}\)

41. \(\sqrt{2}/2\)

43. \(\sqrt{2}\)

45. \(13\sqrt{2}/2\)

47. \(-7\sqrt{2}\)

49. \(3 + \sqrt{3}\)

51. \(-22 - 14\sqrt{2}\)

53. \(-14\)

55. \(-6\)

57. \(9 + 4\sqrt{5}\)

59. \(23 - 4\sqrt{15}\)

61. \(\frac{\sqrt{5} - \sqrt{3}}{2}\)

63. \(\frac{2\sqrt{5} + \sqrt{2}}{3}\)

65. \(7 + 4\sqrt{3}\)

67. \(5 + 2\sqrt{6}\)

69. \((4 \pm \sqrt{31})/3\)

71. \((1 \pm \sqrt{6})/5\)

73. \((3 \pm \sqrt{23})/7\)

75. \(1 \pm 2\sqrt{5}\)

77. \(\frac{1}{\sqrt{x} + \sqrt{2}}\)

79. \(\frac{1}{\sqrt{x + h} + \sqrt{x}}\)
9.5 Exercises

For the rational functions in Exercises 1-6, perform each of the following tasks.

i. Load the function \( f \) and the line \( y = k \) into your graphing calculator. Adjust the viewing window so that all point(s) of intersection of the two graphs are visible in your viewing window.

ii. Copy the image in your viewing window onto your homework paper. Label and scale each axis with xmin, xmax, ymin, and ymax. Label the graphs with their equations. Remember to draw all lines with a ruler.

iii. Use the intersect utility to determine the coordinates of the point(s) of intersection. Plot the point of intersection on your homework paper and label it with its coordinates.

iv. Solve the equation \( f(x) = k \) algebraically. Place your work and solution next to your graph. Do the solutions agree?

1. \( f(x) = \sqrt{x + 3}, \ k = 2 \)

2. \( f(x) = \sqrt{4 - x}, \ k = 3 \)

3. \( f(x) = \sqrt{7 - 2x}, \ k = 4 \)

4. \( f(x) = \sqrt{3x + 5}, \ k = 5 \)

5. \( f(x) = \sqrt{5 + x}, \ k = 4 \)

6. \( f(x) = \sqrt{4 - x}, \ k = 5 \)

In Exercises 7-12, use an algebraic technique to solve the given equation. Check your solutions.

7. \( \sqrt{-5x + 5} = 2 \)

8. \( \sqrt{4x + 6} = 7 \)

9. \( \sqrt{6x - 8} = 8 \)

10. \( \sqrt{2x + 4} = 2 \)

11. \( \sqrt{-3x + 1} = 3 \)

12. \( \sqrt{4x + 7} = 3 \)

For the rational functions in Exercises 13-16, perform each of the following tasks.

i. Load the function \( f \) and the line \( y = k \) into your graphing calculator. Adjust the viewing window so that all point(s) of intersection of the two graphs are visible in your viewing window.

ii. Copy the image in your viewing window onto your homework paper. Label and scale each axis with xmin, xmax, ymin, and ymax. Label the graphs with their equations. Remember to draw all lines with a ruler.

iii. Use the intersect utility to determine the coordinates of the point(s) of intersection. Plot the point of intersection on your homework paper and label it with its coordinates.

iv. Solve the equation \( f(x) = k \) algebraically. Place your work and solution next to your graph. Do the solutions agree?

13. \( f(x) = \sqrt{x + 3} + x, \ k = 9 \)

14. \( f(x) = \sqrt{x + 6} - x, \ k = 4 \)

15. \( f(x) = \sqrt{x - 5} - x, \ k = -7 \)

16. \( f(x) = \sqrt{x + 5} + x, \ k = 7 \)

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In Exercises 17-24, use an algebraic technique to solve the given equation. Check your solutions.

17. \( \sqrt{x + 1} + x = 5 \)
18. \( \sqrt{x + 8} - x = 8 \)
19. \( \sqrt{x + 4} + x = 8 \)
20. \( \sqrt{x + 8} - x = 2 \)
21. \( \sqrt{x + 5} - x = 3 \)
22. \( \sqrt{x + 5} + x = 7 \)
23. \( \sqrt{x + 9} - x = 9 \)
24. \( \sqrt{x + 7} + x = 5 \)

For the rational functions in Exercises 25-28, perform each of the following tasks.

i. Load the function \( f \) and the line \( y = k \) into your graphing window calculator. Adjust the viewing window so that all point(s) of intersection of the two graphs are visible in your viewing window.

ii. Copy the image in your viewing window onto your homework paper. Label and scale each axis with \( \text{xmin}, \text{xmax}, \text{ymin}, \text{ymax} \). Label the graphs with their equations. Remember to draw all lines with a ruler.

iii. Use the intersect utility to determine the coordinates of the point(s) of intersection. Plot the point of intersection on your homework paper and label it with its coordinates.

iv. Solve the equation \( f(x) = k \) algebraically. Place your work and solution next to your graph. Do the solutions agree?

25. \( f(x) = \sqrt{x - 1} + \sqrt{x + 6}, k = 7 \)
26. \( f(x) = \sqrt{x + 2} + \sqrt{x + 9}, k = 7 \)

27. \( f(x) = \sqrt{x + 2} + \sqrt{3x + 4}, k = 2 \)
28. \( f(x) = \sqrt{6x + 7} + \sqrt{3x + 3}, k = 1 \)

In Exercises 29-40, use an algebraic technique to solve the given equation. Check your solutions.

29. \( \sqrt{x + 46} - \sqrt{x - 35} = 1 \)
30. \( \sqrt{x - 16} + \sqrt{x + 16} = 8 \)
31. \( \sqrt{x - 19} + \sqrt{x - 6} = 13 \)
32. \( \sqrt{x + 31} - \sqrt{x + 12} = 1 \)
33. \( \sqrt{x - 2} - \sqrt{x - 49} = 1 \)
34. \( \sqrt{x + 13} + \sqrt{x + 8} = 5 \)
35. \( \sqrt{x + 27} - \sqrt{x - 22} = 1 \)
36. \( \sqrt{x + 40} + \sqrt{x + 13} = 3 \)
37. \( \sqrt{x + 30} - \sqrt{x - 38} = 2 \)
38. \( \sqrt{x + 36} - \sqrt{x + 11} = 1 \)
39. \( \sqrt{x - 17} + \sqrt{x + 3} = 10 \)
40. \( \sqrt{x + 18} + \sqrt{x + 13} = 5 \)
9.5 Answers

1. \( x = 1 \)

3. \( x = -\frac{9}{2} \)

5. \( x = 11 \)

7. \( \frac{1}{5} \)

9. 12

11. \( -\frac{8}{3} \)

13. \( x = 6 \)

15. \( x = 9 \)

17. 3

19. 5

21. \( -1 \)
23. $-8, -9$

25. $x = 10$

27. $x = -1$

29. 1635

31. 55

33. 578

35. 598

37. 294

39. 33
9.6 Exercises

In Exercises 1-8, state whether or not the given triple is a Pythagorean Triple. Give a reason for your answer.

1. $(8, 15, 17)$
2. $(7, 24, 25)$
3. $(8, 9, 17)$
4. $(4, 9, 13)$
5. $(12, 35, 37)$
6. $(12, 17, 29)$
7. $(11, 17, 28)$
8. $(11, 60, 61)$

In Exercises 9-16, set up an equation to model the problem constraints and solve. Use your answer to find the missing side of the given right triangle. Include a sketch with your solution and check your result.

9.

10. 

11. 

12. 

13.

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14. \[12 \quad 4\sqrt{3}\]

15. \[5 \quad 10\]

16. \[8\sqrt{2} \quad 8\]

In Exercises 17-20, set up an equation that models the problem constraints. Solve the equation and use the result to answer the question. Look back and check your result.

17. The legs of a right triangle are consecutive positive integers. The hypotenuse has length 5. What are the lengths of the legs?

18. The legs of a right triangle are consecutive even integers. The hypotenuse has length 10. What are the lengths of the legs?

19. One leg of a right triangle is 1 centimeter less than twice the length of the first leg. If the length of the hypotenuse is 17 centimeters, find the lengths of the legs.

20. One leg of a right triangle is 3 feet longer than 3 times the length of the first leg. The length of the hypotenuse is 25 feet. Find the lengths of the legs.

21. Pythagoras is credited with the following formulae that can be used to generate Pythagorean Triples.

\[
a = m \\
b = \frac{m^2 - 1}{2} \\
c = \frac{m^2 + 1}{2}
\]

Use the technique of Example 6 to demonstrate that the formulae given above will generate Pythagorean Triples, provided that \(m\) is an odd positive integer larger than one. Secondly, generate at least 3 instances of Pythagorean Triples with Pythagoras’s formula.

22. Plato (380 BC) is credited with the following formulae that can be used to generate Pythagorean Triples.

\[
a = 2m \\
b = m^2 - 1 \\
c = m^2 + 1
\]

Use the technique of Example 6 to demonstrate that the formulae given above will generate Pythagorean Triples, provided that \(m\) is a positive integer larger than 1. Secondly, generate at least 3 instances of Pythagorean Triples with Plato’s formula.
In Exercises 23-28, set up an equation that models the problem constraints. Solve the equation and use the result to answer the question. Look back and check your result.

23. Fritz and Greta are planting a 12-foot by 18-foot rectangular garden, and are laying it out using string. They would like to know the length of a diagonal to make sure that right angles are formed. Find the length of a diagonal. Approximate your answer to within 0.1 feet.

24. Angelina and Markos are planting a 20-foot by 28-foot rectangular garden, and are laying it out using string. They would like to know the length of a diagonal to make sure that right angles are formed. Find the length of a diagonal. Approximate your answer to within 0.1 feet.

25. The base of a 36-foot long guy wire is located 16 feet from the base of the telephone pole that it is anchoring. How high up the pole does the guy wire reach? Approximate your answer to within 0.1 feet.

26. The base of a 35-foot long guy wire is located 10 feet from the base of the telephone pole that it is anchoring. How high up the pole does the guy wire reach? Approximate your answer to within 0.1 feet.

27. A stereo receiver is in a corner of a 13-foot by 16-foot rectangular room. Speaker wire will run under a rug, diagonally, to a speaker in the far corner. If 3 feet of slack is required on each end, how long a piece of wire should be purchased? Approximate your answer to within 0.1 feet.

28. A stereo receiver is in a corner of a 10-foot by 15-foot rectangular room. Speaker wire will run under a rug, diagonally, to a speaker in the far corner. If 4 feet of slack is required on each end, how long a piece of wire should be purchased? Approximate your answer to within 0.1 feet.

In Exercises 29-38, use the distance formula to find the exact distance between the given points.

29. \((-8, -9)\) and \((6, -6)\)
30. \((1, 0)\) and \((-9, -2)\)
31. \((-9, 1)\) and \((-8, 7)\)
32. \((0, 9)\) and \((3, 1)\)
33. \((6, -5)\) and \((-9, -2)\)
34. \((-9, 6)\) and \((1, 4)\)
35. \((-7, 7)\) and \((-3, 6)\)
36. \((-7, -6)\) and \((-2, -4)\)
37. \((4, -3)\) and \((-9, 6)\)
38. \((-7, -1)\) and \((4, -5)\)

In Exercises 39-42, set up an equation that models the problem constraints. Solve the equation and use the result to answer the question. Look back and check your result.

39. Find \(k\) so that the point \((4, k)\) is \(2\sqrt{2}\) units away from the point \((2, 1)\).

40. Find \(k\) so that the point \((k, 1)\) is \(2\sqrt{2}\) units away from the point \((0, -1)\).
41. Find \( k \) so that the point \((k,1)\) is \(\sqrt{17}\) units away from the point \((2,-3)\).

42. Find \( k \) so that the point \((-1,k)\) is \(\sqrt{13}\) units away from the point \((-4,-3)\).

43. Set up a coordinate system on a sheet of graph paper. Label and scale each axis. Plot the points \(P(0,5)\) and \(Q(4,-3)\) on your coordinate system.

   a) Plot several points that are equidistant from the points \(P\) and \(Q\) on your coordinate system. What graph do you get if you plot all points that are equidistant from the points \(P\) and \(Q\)? Determine the equation of the graph by examining the resulting image on your coordinate system.

   b) Use the distance formula to find the equation of the graph of all points that are equidistant from the points \(P\) and \(Q\). Hint: Let \((x,y)\) represent an arbitrary point on the graph of all points equidistant from points \(P\) and \(Q\). Calculate the distances from the point \((x,y)\) to the points \(P\) and \(Q\) separately, then set them equal and simplify the resulting equation. Note that this analytical approach should provide an equation that matches that found by the graphical approach in part (a).

44. Set up a coordinate system on a sheet of graph paper. Label and scale each axis. Plot the point \(P(0,2)\) and label it with its coordinates. Draw the line \(y=-2\) and label it with its equation.

   a) Plot several points that are equidistant from the point \(P\) and the line \(y=-2\) on your coordinate system. What graph do you get if you plot all points that are equidistant from the points \(P\) and the line \(y=-2\)?

   b) Use the distance formula to find the equation of the graph of all points that are equidistant from the points \(P\) and the line \(y=-2\). Hint: Let \((x,y)\) represent an arbitrary point on the graph of all points equidistant from points \(P\) and the line \(y=-2\). Calculate the distances from the point \((x,y)\) to the points \(P\) and the line \(y=-2\) separately, then set them equal and simplify the resulting equation.
45. Copy the following figure onto a sheet of graph paper. Cut the pieces of the first figure out with a pair of scissors, then rearrange them to form the second figure. Explain how this proves the Pythagorean Theorem.

46. Compare this image to the one that follows and explain how this proves the Pythagorean Theorem.
9.6 Answers

1. Yes, because \(8^2 + 15^2 = 17^2\)
3. No, because \(8^2 + 9^2 \neq 17^2\)
5. Yes, because \(12^2 + 35^2 = 37^2\)
7. No, because \(11^2 + 17^2 \neq 28^2\)
9. 4
11. \(4\sqrt{3}\)
13. \(2\sqrt{2}\)
15. \(5\sqrt{3}\)

17. The legs have lengths 3 and 4.
19. The legs have lengths 8 and 15 centimeters.
21. \((3, 4, 5), (5, 12, 13),\) and \((7, 24, 25)\), with \(m = 3, 5,\) and 7, respectively.
23. 21.63 ft
25. 32.25 ft
27. 26.62 ft
29. \(\sqrt{205}\)
31. \(\sqrt{37}\)
33. \(\sqrt{234} = 3\sqrt{26}\)
35. \(\sqrt{17}\)
37. \(\sqrt{250} = 5\sqrt{10}\)
39. \(k = 3, -1\).
41. \(k = 1, 3\).

43. (a) In the figure that follows, \(XP = XQ\).

(b) \(y = (1/2)x\)