Exercise 5: Orbits and gravity

Learning objective: To draw the least energy interplanetary trajectory of a spacecraft, and to determine the speed and energy needed for such a trajectory.

Needed: Graph paper, compass, maybe a ruler, definitely a calculator

Though a straight line may be the shortest distance between two points, travelling along the straight line requires a huge expenditure of energy, especially between two planets. In fact, the lowest energy path between two planets is along a **Hohmann transfer orbit**, named for the German engineer Walter Hohmann, who first described this orbit in his 1925 book *The Attainability of Celestial Bodies*. This orbit is best described in a diagram which you will draw, showing Earth and Jupiter. The ballistic orbit for a trip to Jupiter from Earth would have its **perihelion** on Earth's orbital path (1 AU) and its **aphelion** on Jupiter's orbital path (5.25 AU).

1. On a separate sheet of paper, draw a point at the center of the sheet. This will represent the Sun. Make the assumption that planetary orbits are circular (in this approximation, it's an okay assumption) and, using a scale of 1 AU = 1 inch), draw the orbit of Earth at 1 AU and the orbit of Jupiter at 5.25 AU (all of the orbit won’t fit on the paper). At the end of the exercise, don’t forget to *attach* your drawing.

2. What is the **shape** of a Hohmann transfer orbit (hint: think Kepler's first law)?

3. Next, **draw** the probe's Hohmann transfer orbit (make it a smooth shape) between Earth's orbit and Jupiter's orbit and highlight the portion of the orbit the probe will use between Earth and Jupiter.

4. What is the **period** of the probe's orbit (hint: use Kepler's third law)? But how long will the **actual flight time** of the probe be (look at the highlighted portion of the orbit)?

5. **Draw** a circle representing Earth where the probe's orbit intersects Earth's orbit and **draw** another circle representing Jupiter where the probe's orbit intersects Jupiter's orbit. You will draw one more Earth and one more Jupiter, for a total of four bodies drawn. They will be labelled "Earth at probe launch", "Jupiter at probe launch", "Earth at probe landing" and "Jupiter at probe landing". Label the Earth and Jupiter you have already drawn with the correct labels.
6. (Payoff question) Where should Jupiter be when the probe is launched? **Draw and label it on your diagram.** About **how much of one orbit** (in degrees) will Jupiter have moved during the probe flight? Show your calculation.

7. a. Where will Earth be when the probe lands? **Draw and label it on your diagram.** Approximately **how much of one orbit** (in degrees) will the Earth have moved while the probe is in flight?

b. Will the **Sun** get in the way of **radio communications** with the probe? How can you tell?

The **vis-viva** ("living force") equation was derived from the principle of conservation of energy (most attribute this to the work of Gaspard-Gustave Lavoisier and Pierre-Simon Laplace in 1783, about a century after Newton publishes *Principia*) and connects the speed of an orbiting body at any given point with the distance that it is away from the body it is orbiting. It was

\[ v^2 = G \left( m_1 + m_2 \right) \left( \frac{2}{r} - \frac{1}{a} \right) \]

where:
- \( v \) is the speed of the orbiting body in m/s
- \( G = 6.67 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2 \)
- \( m_1 \) is the mass of the orbiting object in kg
- \( m_2 \) is the mass of the object being orbited in kg
- \( r \) is the instantaneous distance between the two objects in m
- \( a \) is the semi-major axis of the orbiting object in m
8. Use the vis-viva equation to calculate the orbital speed needed to keep a satellite with a mass of 500 kg in a **circular** orbit around the earth at a distance of 200 km.

   a. For this probe, what **simplification** can you make to the sum \((m_1 + m_2)\)? Hint: consider what \(m_1\) and \(m_2\) represent.

   b. Calculate the orbital **speed** (in m/s, then km/s) needed for the probe.

9. a. For the Jupiter probe earlier in the exercise, what object is represented by \(m_2\)? So what **simplification** can you make to the sum \((m_1 + m_2)\)?

   b. For the Jupiter probe leaving Earth orbit, what is \(r\) (in AU, then in m)? What is \(a\) (in AU, then in m)?
10. Using the \textit{vis-viva} equation, what \textbf{speed} (in km/s) must the Jupiter probe achieve in Earth orbit in order to make it to Jupiter? Show the calculation! Given that the Earth is already orbiting the Sun at a speed of 29.8 km/s, will the Earth’s orbital speed and the orbital speed of the spacecraft be enough to push the spacecraft into an orbit for Jupiter? Indicate these speeds on your orbital diagram with arrows. If there is not enough speed, calculate $\Delta v$ (“delta-vee”), the additional speed the thrusters on the spacecraft will have to supply.

11. Of course, launching a “miniaturized” probe of 500 kg is different than launching something as large as Mars Science Laboratory, which weighed 500,000 kg. Using the formula $E = \frac{1}{2} m v^2$, where $E$ is energy needed to move an object with mass $m$ and speed $v$, calculate the \textbf{amount of energy needed to launch} the “miniaturized” probe versus the 500,000 kg spacecraft. Your answer will be in Joules (J), if you start with mass in kg and speed in m/s.
12. What is Jupiter’s **orbital speed** (in m/s, then km/s)? You can calculate this by finding Jupiter’s orbital circumference (assume the orbit is circular for now) and Jupiter’s orbital period, and then applying the correct operation.

13. Finally, use the vis-viva equation to calculate the probe’s speed when it approaches Jupiter (in m/s, then km/s).

14. Payoff: Will the probe need to slow down or will it need to speed up once it reaches Jupiter? Remember, for “**orbital insertion**”, the probe should have the same speed as Jupiter in the same direction that Jupiter is moving. How much does it need to slow down or speed up by? Note that this is why every probe does need to carry an on-board fuel supply!