EE 215A Fundamentals of Electrical Engineering
Lecture Notes

Circuit Basics

7/27/01 - last review 9/28/04

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Introduction:

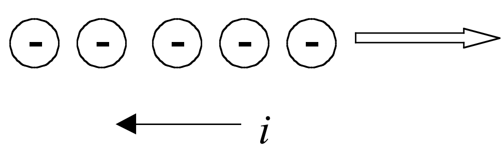
Electrical Engineering (in grotesque oversimplification) is about figuring out what current and voltage are in a circuit (*analysis*) - OR - figuring out how to get current and voltage to do what you want them to do (*synthesis*, aka *design*).

So what the heck are *current* and *voltage*? (You already know from Physics, but let's assume it's been a while…)

Current:

Electrical *current* is the movement, or flow, of *charge*. Just like current in a river is the movement of water.

In most circuits what moves are electrons, which have a negative charge. The movement of negative charge in one direction corresponds to the flow of positive current in the opposite direction.



(The reason for this is that Ben Franklin had to pick a direction for current flow to be positive. He had no idea what actually moved. He guessed wrong.)

Don't worry too much about what physically moves until you are inside the guts of semiconductors in advanced EE courses. Just think of current as the flow of positive charge.

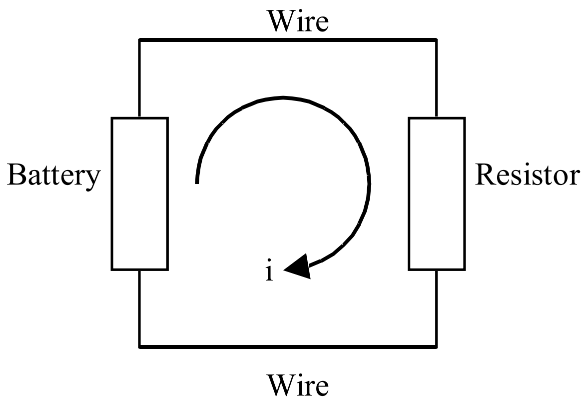
Charge, q, is measured in coulombs, C.

Current, i, is measured in amperes, A. (Named after a French physicist.)

Current is the change in charge with time:

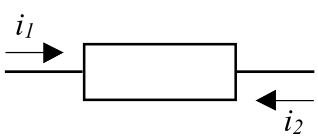
$$i = \frac{dq}{dt} \qquad 1A = \frac{1\,\text{coul}}{1\,\text{sec}}$$

Current must flow in a loop (hence, *circuit*). Charge cannot pile up in anywhere (law of conservation of charge).



In this elementary circuit, current flows around the circuit in a loop.

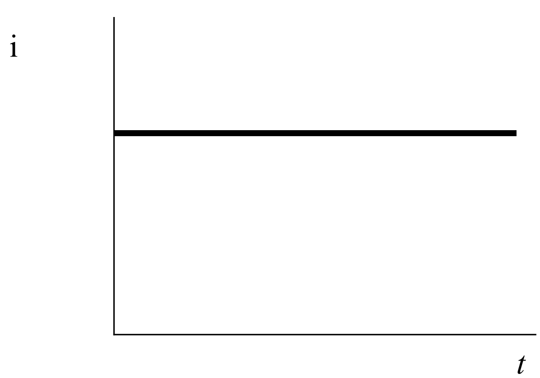
In a *circuit element* (the generic name for something you stick in a circuit), current flowing in must equal current flowing out. (Similarly, charge in equals charge out.)



$i_1 = - i_2$

Current comes in two major forms:

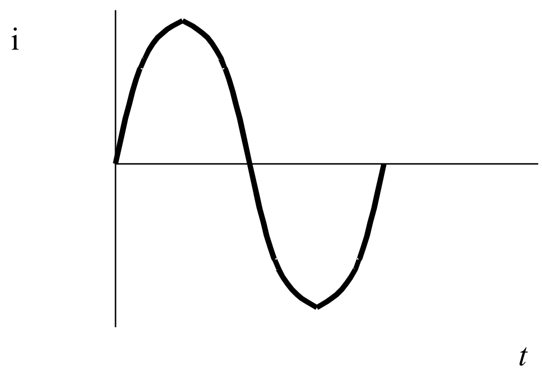
Direct Current (dc): i is constant



dc current is constant with respect to time. The mathematical representation is, for example, $i = 5\text{A}$.

The other major form is

Alternating Current (ac): i is sinusoidal

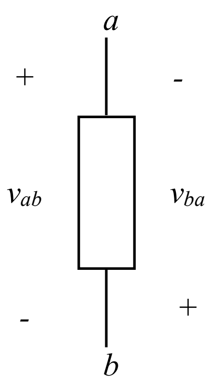


ac current is sinusoidal with respect to time. The mathematical representation is, for example, $i = 5\cos 377t$ A.

Voltage:

Like pressure making water flow, creating a current, *voltage* makes charge flow, creating a current. Voltage is electrical pressure. Voltage has symbol v and units of volts (V), named after an Italian physicist. (Volta of frog leg fame.)

Pressure is measured between two points, a high pressure point and a low pressure point. (Even when you measure the pressure in a tire, you are measuring the pressure between the tire and the atmosphere.)



Similarly, voltage is measured between two points, a high point (+) and a low point (-). Knowing which point is the high point and which the low is called *polarity*. (Direction of positive flow is current polarity.) Often the points are given letters or numbers to identify them. These can then be used in subscripts.

When two subscripts are used, the first subscript is the positive point of measurement (high) and the second is the negative (low).

Many times only one subscript is used. This is the first subscript. The second is understood to be a common reference point. We do this with pressure a lot. For example, if I have 30 pounds of pressure in my tire, it is really tire pressure minus air pressure - the air pressure is the understood reference.

Power:

Another definition of voltage, which is consistent with calling it electrical pressure, is that voltage is the amount of *work* (which is *energy*, of course) needed to move one unit of charge from the negative to the positive terminal. Mathematically,

$$v = \frac{dW}{dq}$$

where W is work. Voltage is an MKS unit, so moving 1 coulomb across 1 volt requires 1 joule of energy.

A lot of the general public (like news people) get *power* and energy mixed up. Power, p, is the change in energy with time.

$$p = \frac{dW}{dt}$$

Let's do a trick and apply the Chain Rule.

$$p = \frac{dW}{dq}\frac{dq}{dt}$$

Now note that we know what each of the derivatives in the chain is: voltage and current! So

$$\boxed{p = v \cdot i}$$

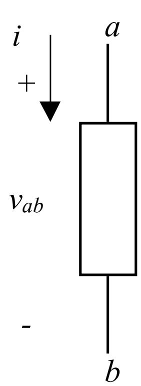
Power equals voltage times current is a fundamental electrical relationship.
Power is measured in watts, W. (Named after a Scottish steam engine maker.)

Flow of 1 A through 1 V requires power of 1 W. This must be flow from the negative terminal to the positive terminal.

Now, there are two possible ways that current and voltage polarities can exist for one circuit element. Current can flow into the positive voltage end (and therefore, out of the negative end) or current can flow out of the positive end (and therefore, in to the negative end.) The polarity relationship determines whether power is flowing from the environment to the circuit (moving current from - to +) or from the circuit to the environment (current moving from + to -).

EE's regularly use a number of different phrases when talking about the polarity relationship. This can be very confusing at first, because there are so many and sometimes the meaning is not immediately obvious. But the phrases must be useful in the long run or they would not be in common use.

Here are the two relationships, and some ways EEs talk about them:

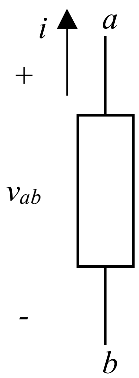


Passive Sign Convention

Current flows into the positive end.
Power flows from the circuit to the environment.
Power is absorbed by the element (from the circuit)
Power is dissipated by the element (from the circuit, to the environment)
Power is supplied to the element (from the circuit)
Power flows into the element (from the circuit)

Typical for a resistor.

Not Passive Sign Convention



Current flows out of the positive end.
Power flows from the environment into the circuit.
Power is supplied by the element (to the circuit)
Power is delivered by the element (to the circuit).
Power flows out of the element (to the circuit).

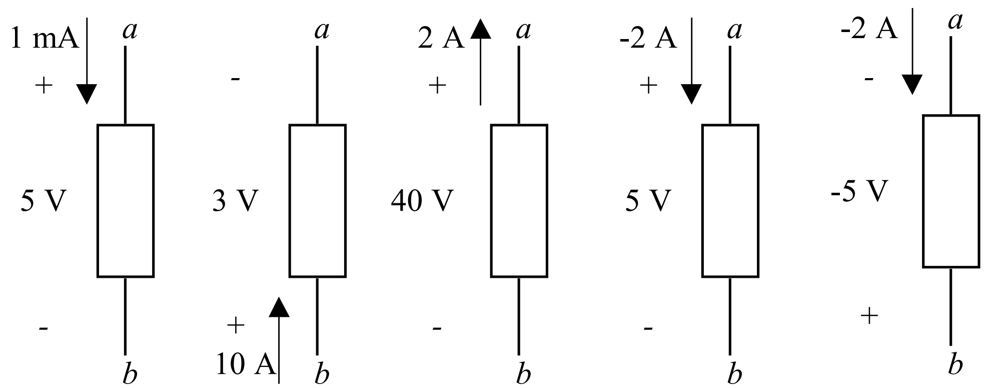
Typical for a source.

As you can see, some of the differences are quite subtle (supplied by the circuit, supplied to the circuit).

Also, what's with the "Not Passive Sign Convention"? In general, circuit elements that absorb power (like resistors) are called passive elements, and circuit elements that supply power (to the circuit!) are called active elements. But for some reason, the opposite of the passive sign convention is not called the active sign convention. (Of course, people use the term active sign convention, but they are not supposed to. If you want to be an alpha geek, find out why and go around correcting them!) Instead, the awkward term "Not passive sign convention" is used. Actually, the full blown correct expression is

"The element does not conform to the passive sign convention." But that's way too long to say or write (unless you are being the alpha geek).

The positive end is the positive end, even if the voltage is negative. The direction of current flow is the same whether current is positive or negative. (Of course, negative current flowing one way is positive current flowing the other.) And negative power can be absorbed if you have a passive sign convention. The polarity determines the sign convention, not the signs of the voltage or current. Here's some practice. For each element, determine whether it is passive or not passive, find the power, and state whether the power is absorbed or supplied by the element.



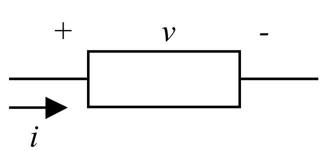
The first one is passive, p = 5 mW, absorbed by element.
The second is passive, p = 30 W, absorbed by element
The third is not passive (current flow would be in to negative terminal), p = 80 W, supplied by element.
The fourth is passive, p = -10 W, absorbed by element. Note that -10W absorbed is the same as 10 W supplied by the element.
The fifth is about as complicated as I can make it and is left as an exercise for the student!

Review:

Current is flow of charge $i = \frac{dq}{dt}$ in amperes A
Voltage is electrical pressure, is work done to move electron from (-) to (+) terminal
$v = \frac{dw}{dq}$ in volts V
Power $\boxed{p = v \cdot i}$ in watts W

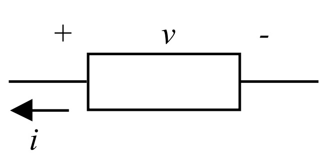
Passive sign convention:

Power is dissipated by circuit element
Resistors



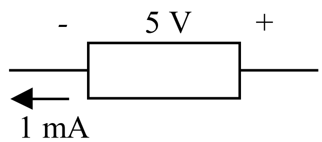
NOT the passive sign convention

Power is supplied by the element to the circuit

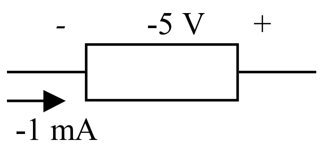


Exercise:

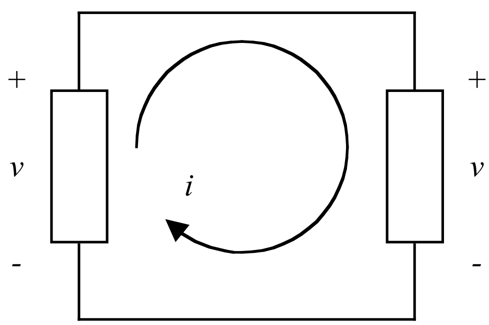
Passive or not? Find p.



Passive or not? Find p.



Energy Flow and Sign Convention

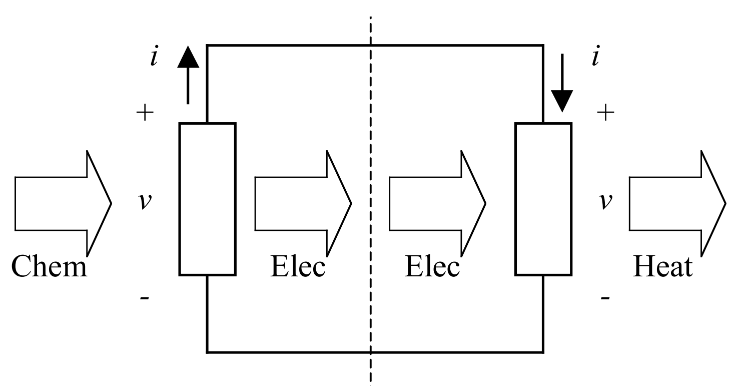


Consider these two elements.
The same voltage appears across each of them.
Current must flow in the circuit (assuming charge does not accumulate in one of the elements.)

Which is passive, which active?

The left element does NOT adhere to the passive sign convention.

Energy (shown as the broad arrow) comes in from the environment and does work to move charge from the negative to the positive terminal (the original definition of voltage).



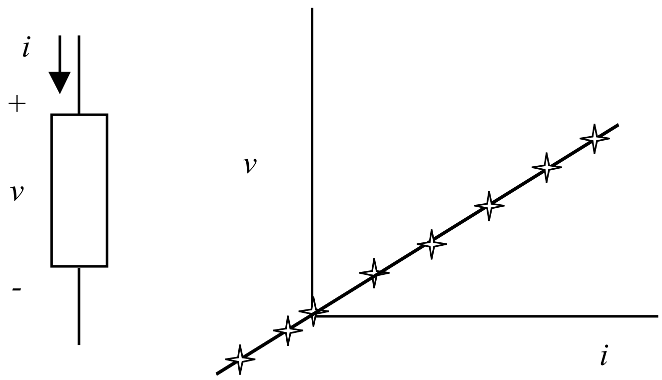
This energy then flows into the circuit. Not passive means the positive sign of energy comes in from the environment and flows to the circuit. A battery is a good example, it converts energy from chemical to electrical. In water flow analogy, a pump takes energy from its shaft and converts it to the energy in water flow.

The right element adheres to the passive sign convention. Energy flows in to it from the circuit and is dissipated to the environment. A resistor is the most typical passive element, it converts electric energy to heat energy. The water flow analogy is to head loss in long pipes. The friction of the water appears as heat. (NOT the conductive heat transfer in a heat exchanger!)

Unless there is energy storage, energy in equals energy out.

Resistance and Ohm's Law

Speaking of resistors, suppose we took a circuit element adhering to the passive sign convention, varied the voltage and plotted the current, and this was what we saw:



Just for fun, let's take a few more data points with negative voltages.

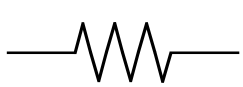
When we see the points fall on a straight line passing through the origin, we know what to write:

$$v = ki$$

We call the constant of proportionality **resistance** R, measured in Ohms, symbol Ω

$\boxed{v = iR}$ is Ohm's Law, another fundamental EE relationship

If R is constant, we say the element is linear. (There is a stiffer definition of linearity in the book.)



The standard resistor symbol has zig-zag lines to indicate that it is harder to push current through.

From the graph, we could have chosen to say

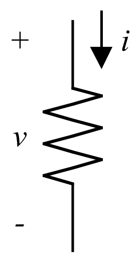
$$i = kv$$

The constant here is called **conductance**, G, measured in siemens under the SI units of measurement. Older or whimsical engineers still give it units of mhos, inverse ohms, drawn with an inverted capital omega absent from most computer character sets!

$$i = Gv$$

$$G = \frac{1}{R}$$

Resistor polarities follow the passive sign convention:



Power absorbed by the resistor appears as heat

$$p = vi$$

$$p = (iR)i$$

$$\boxed{p = i^2 R}$$

$$p = vi$$

$$p = v\frac{v}{R}$$

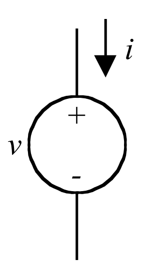
$$\boxed{p = \frac{v^2}{R}}$$

The resistor power dissipation formulas are two more fundamental relationships in EE. (Don't worry, we're about done with them. I think.)

Example:

10 Ω resistor has 2 A flowing through it. How much power does it dissipate? p = 40 W
(Note: don't need to know which direction!)
10 Ω resistor has 20 V across it. How much power is dissipated? p = 40 W
(Again, don't need to know polarity.)
10 siemen resistor has 20 V across it. $p = Gv^2$ = 4000 W = 4 kW

Sources

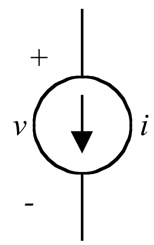


A **voltage source** is a circuit element that controls the voltage across it. The current through a voltage source is determined by the rest of the circuit.

The polarity of the voltage usually appears in the circle. (Empty circles are voltage sources.)

A battery is a typical voltage source.

The water flow analogy is a constant pressure pump (e.g. centrifugal flow).



A current source is a circuit element that controls the current flowing through it. The voltage across a current source is determined by the rest of the circuit.

The direction of current flow always appears in the circle.

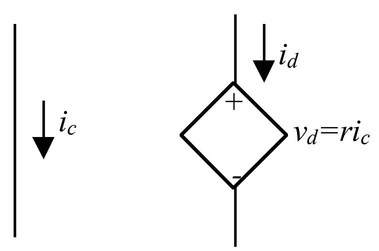
There are no common current sources, and ones that approach ideal behavior are hard to build and expensive.

The water flow analogy is a constant flow pump (e.g. positive displacement).

Independent sources control the value of voltage or current independently of what goes on in the rest of the circuit. (Remember that only one of voltage or current is controlled by a source.)

Dependent sources control the value of voltage or current as a function of voltage or current elsewhere in the circuit. While the voltage or current is controlled by this function, the other electrical parameter, current or voltage, respectively, is still determined by the rest of the circuit.

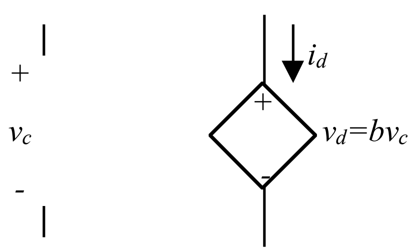
If you think about it, there are four basic types of dependent source:



CCVS, Current Controlled Voltage Source

Source voltage v_d is a linear function of a control current i_c flowing somewhere else in the circuit. r is called the gain.

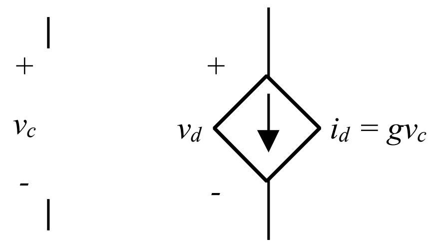
(The function does not have to be linear!)



VCVS, Voltage Controlled Voltage Source

Source voltage v_d is a linear function of a control voltage v_c measured somewhere else in the circuit. b is the gain.

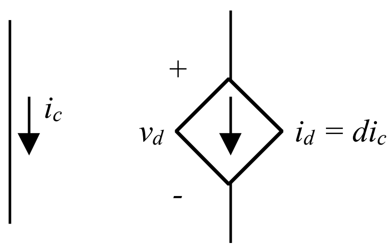
Op amps are modeled with VCVSs.



VCCS, Voltage Controlled Current Source

Source current i_d is a linear function of a control voltage v_c measured somewhere else in the circuit. g is the gain.

Transistors can be modeled with VCCSs.

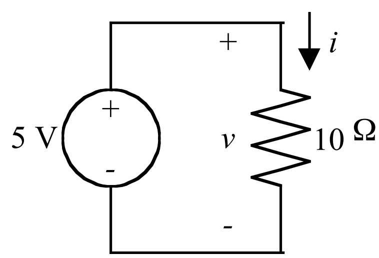


CCCS, Current Controlled Current Source

Source current i_d is a linear function of a control current i_c flowing somewhere else in the circuit. d is the gain.

Transistors can also be modeled with CCCSs.

Let's look at examples

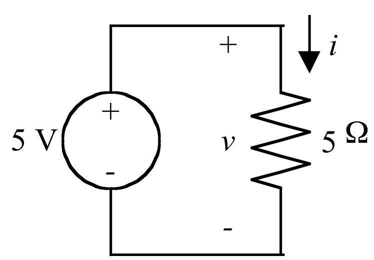


Find v - held to 5 V by source.

Find i = 5 V/10 Ω = 0.5 A by Ohm's Law

Find $p_R = i^2R$ = 2.5 W dissipated by R

Find $p_s = vi$ = 2.5 W supplied to circuit (checks)



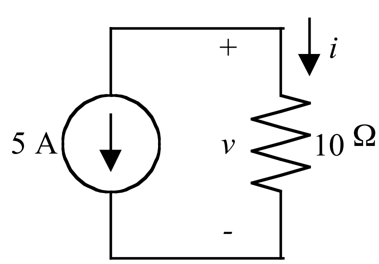
Find v - held to 5 V by source.

Find i = 5 V/5 Ω = 1 A by Ohm's Law

Find $p_R = i^2R$ = 5 W dissipated by R

Find $p_s = vi$ = 5 W supplied to circuit (checks)

Note source voltage does not change. Current changes because resistance changes. Shows how uncontrolled value (current or voltage) depends on rest of circuit.



Find $v = iR$ = 50 V + or -? (-)

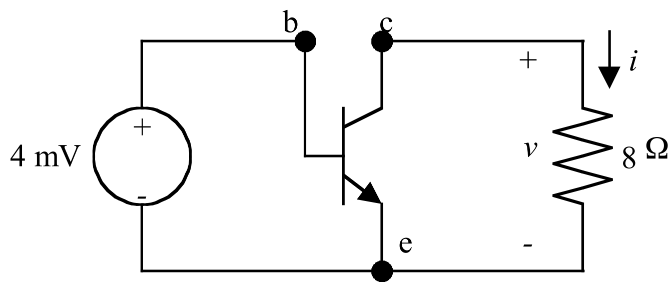
Find $i = -i_s = -5$ A

Find $p_R = i^2R = 250$ W dissipated by R

Find $p_s = vi = -250$ W dissipated by source
(which is, hmm, 250 W supplied by source)
(checks)

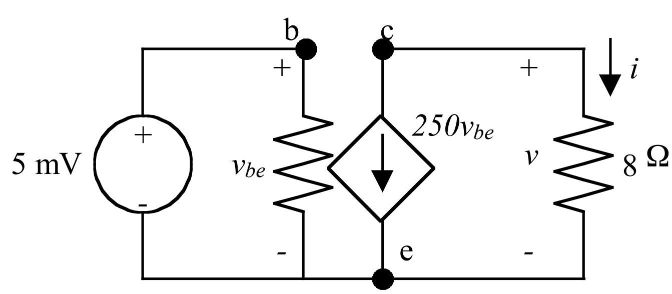
Let's try a transistor model.

Here is an (incomplete) transistor amplifier circuit:



The funny looking symbol is a transistor, a semiconductor device with three points of connection called the emitter, base and collector. We're going to try to amplify a small input voltage, maybe from an antenna, into a signal you can hear on an 8 ohm speaker. We'll do that with a model of the transistor. (You are not responsible for the models in this course, just for analyzing them once I give them to you.) (By the way, this is not so good a model for the transistor and a resistor is not so good a model for a speaker. We'll work towards better (more complicated) models.)

Our model replaces the transistor with a resistor and a current controlled voltage source:



(I can't guarantee that the value $g_m = 250$ is correct.)

The first thing to note in this circuit is that the value of the resistor in the transistor model does not affect the analysis, since v_{be} will be equal to the source voltage 5 mV (unless there is a short circuit).

Next, note that the current produced by the dependent source can only flow through the output resistor (8 Ω).

Next, in the generic VCCS, the control voltage was measured across an open circuit, and here it is measured across a resistor. The point is that the measured voltage is between any two points, whether anything is attached to the points or not.

Analysis:

$i_d = 250 v_{be}$ = 250 (5 mV) = 1250 mA = 1.25 A.

$i = -\ i_d = -1.25$ A

$v = i\mathrm{R} = -1.25 \times 8 = -10$ V

$p = vi$ = 12.5 W (obviously all from dependent source) dissipated by resistor.